

EXPONENTIALLY EXPANDING RADIATION DOMINATED AND DUST DOMINATED UNIVERSES IN BRANS-DICKE THEORY

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The Brans-Dicke Theory of Gravity is one of the most promising alternatives to the Einstein's Theory of General Relativity. We have examined an action term with wrong signs for both the kinetic and mass terms for the scalar field and have found solutions for both the scale factor of the universe and the Brans-Dicke scalar field which vary exponentially in time.

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Recent observations of Type Ia supernovae [1]-[2] and measurements of the cosmic microwave background [3] suggests that the universe is in an accelerating expansion phase [4]. The universe seems to be flat [5]-[7], with energy content, $\Omega_{dust} = 0.25$ which contains baryonic matter and CDM with pressure $P_{dust} = 0$ [8]-[11]. The universe seems predominantly filled with dark energy or quintessence which drives the expansion by its negative pressure [12]-[14]. In this paper we will assume that the universe is filled homogeneously by quintessence, the scalar field, ϕ .

The Brans-Dicke Theory of Gravity is one of the leading alternatives to Einstein's Theory of General Relativity [15]. It may be said that Einstein's Theory of General Relativity (GR) is a special case of Brans-Dicke Theory. As the dimensionless constant ω , of Brans-Dicke Theory approaches infinity, the theory approximates GR [16].

We will model the universe with Brans-Dicke Theory with the scale factor varying exponentially with time, that is $a(t) = e^{Ht}$. Let us consider a simple minded model where the relationship between the scalar field ϕ and the energy density of rest of the matter ρ_M is given as,

$$\frac{1}{2}\dot{\phi}^2 + \frac{1}{2}m^2\phi^2 = \rho_M. \quad (1)$$

We are assuming the quintessence, the scalar field tracks the matter energy density ρ_M according to this equation. If there's an exponential solution for the scalar field such that $\phi(t) = e^{Ft}$ the above equation will become

$$\frac{1}{2}(F^2 + m^2)\phi^2 = \rho_M. \quad (2)$$

We will have a linear relationship between the ϕ^2 and ρ_M terms. We further assume the scale factor of the universe varies exponentially with respect to time, $a(t) = e^{Ht}$. For dust dominated universe $\rho_{dust} \sim 1/a^3$, so we get,

$$\rho_{dust} \sim \frac{1}{a^3} = e^{-3Ht}. \quad (3)$$

From (2) we get

$$\phi \sim e^{-3/2Ht}, \quad (4)$$

so F is equal to,

$$F = -3/2H. \quad (5)$$

Similarly for the radiation dominated universe $\rho_{radiation} \sim 1/a^4$, so we get,

$$\rho_{radiation} \sim \frac{1}{a^4} = e^{-4Ht} \quad (6)$$

and

$$\phi \sim e^{-2Ht}, \quad (7)$$

$$F = -2H. \quad (8)$$

We will see that our rigorous results obtained using Brans-Dicke theory will match these values. We thus consider the action

$$S = \int d^4x \sqrt{g} \left(-\frac{1}{8\omega}\phi^2 R - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + \frac{1}{2}m^2\phi^2 + L_M \right). \quad (9)$$

The signature of the metric is $(+,-,-,-)$. The kinetic and the mass terms of the scalar field are negative of the action of the standard Brans-Dicke Theory, i.e. in field theory terminology the ϕ -field is a ghost. However since both the signs of the kinetic and the mass terms are changed the free ϕ field satisfies the conventional Klein-Gordon equation with plane wave solutions. The fact that there are ghost fields in a theory does not mean that the theory is unphysical. As an example one may mention the Lee-Wick model which includes ghost fields but has unitary S matrix [17].

The variation of this action with respect to Robertson Walker metric gives us the following gravitational field equations;

$$\frac{3}{4\omega}\phi^2 \left(\frac{\dot{a}^2}{a^2} + \frac{k}{a^2} \right) + \frac{1}{2}\dot{\phi}^2 + \frac{1}{2}m^2\phi^2 + \frac{3}{2\omega}\frac{\dot{a}}{a}\dot{\phi}\phi = \rho_M \quad (10)$$

$$\begin{aligned} & -\frac{1}{4\omega}\phi^2 \left(2\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} + \frac{k}{a^2} \right) - \frac{1}{\omega}\frac{\dot{a}}{a}\dot{\phi}\phi - \frac{1}{2\omega}\ddot{\phi}\phi + \\ & + \left(\frac{1}{2} - \frac{1}{2\omega} \right) \dot{\phi}^2 - \frac{1}{2}m^2\phi^2 = P_M \end{aligned} \quad (11)$$

$$\ddot{\phi} + 3\frac{\dot{a}}{a}\dot{\phi} + \left[m^2 + \frac{3}{2\omega} \left(\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} + \frac{k}{a^2} \right) \right] \phi = 0, \quad (12)$$

where k is the curvature parameter with $k = -1, 0$ and 1 corresponding to open, flat and closed universes respectively. $a(t)$ is the scale factor of the universe. Dot denotes derivative with respect to time. M stands for everything except the Brans-Dicke scalar field.

We will solve the above equations for flat universe $k = 0$, and with the scale factor of the universe a and the scalar field ϕ varying exponentially with time. That is; $a(t) = e^{Ht}$ and $\phi(t) = e^{Ft}$. The scalar field, ϕ is independent of space coordinates, it is homogeneously distributed over the universe. The three gravitational field equations above (10, 11, and 12) will simply give;

$$-F^2 - 3HF - \frac{3}{\omega}H^2 = m^2 \quad (13)$$

$$\frac{3}{4\omega}H^2 + \frac{F^2}{2} + \frac{m^2}{2} + \frac{3}{2\omega}HF = \frac{\rho_M}{\phi^2} \quad (14)$$

$$-\frac{3}{4\omega}H^2 - \frac{HF}{\omega} - \frac{F^2}{\omega} + \frac{F^2}{2} - \frac{m^2}{2} = \frac{P_M}{\phi^2}. \quad (15)$$

By substituting (13) into (14), and (15) we get,

$$-\frac{3}{4\omega}H^2 - \frac{3}{2}HF \left(\frac{\omega-1}{\omega} \right) = \frac{\rho_M}{\phi^2} \quad (16)$$

$$\frac{3}{4\omega}H^2 + \frac{3}{2}HF \left(\frac{\omega-1}{\omega} \right) + \frac{1}{2\omega}HF + F^2 \left(\frac{\omega-1}{\omega} \right) = \frac{P_M}{\phi^2}. \quad (17)$$

For $\rho_M > 0$ (16) gives

$$-F > \frac{H}{2(\omega-1)} \quad (18)$$

this constraint (18) also satisfies $P_M + \rho_M > 0$ when substituted into (16) and (17).

$\nu = P_M/\rho_M$ can be obtained as;

$$\nu = \frac{P_M}{\rho_M} = -1 + \frac{2(-F)}{3H}. \quad (19)$$

Notice that ν is independent of the Brans-Dicke parameter ω . Since we have assumed an exponential expansion for the universe, in the Einsteinian limit where the gravitational constant does not change, $F = 0$ and one obtains $\nu = -1$ as usual. However for the general Brans-Dicke case one can have exponential expansion for any value of ν provided that the gravitational constant also changes according to this equation.

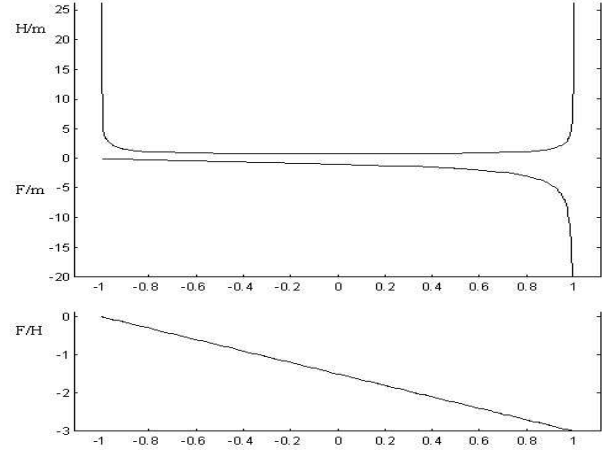


FIG. 1: H/m , F/m and F/H versus ν .

The ratios of H/m and F/m can be found as

$$\frac{2}{3\sqrt{-\nu^2 + (1 - \frac{4}{3\omega})}} = \frac{H}{m} \quad (20)$$

and

$$\frac{\nu + 1}{\sqrt{-\nu^2 + (1 - \frac{4}{3\omega})}} = \frac{-F}{m} \quad (21)$$

for ν sufficiently larger than -1 and sufficiently smaller than 1 . That is, $-\sqrt{1 - 4/3\omega} < \nu < \sqrt{1 - 4/3\omega}$. Time delay experiments give the present lower limit for the constant ω as $\omega > 10^4$ [18]-[20]. So $1/\omega \ll 1$, the $1/\omega$ terms may be ignored.

For a radiation dominated universe $\nu = 1/3$ so that $H = m/\sqrt{2}$ and $F = -\sqrt{2}m$. Whereas for a dust dominated universe $\nu = 0$ so that $H = 2/3m$ and $F = -m$.

We see that the Hubble constant slightly decreases as the universe evolves from the radiation dominated era into the matter dominated era. In fact the minimum value of H is obtained for $\nu = 0$. The Newtonian gravitational constant $G_N \sim \phi^{-2} \sim e^{-2Ft}$, on the other hand increases at a faster rate during radiation dominated era as $-F$ increases as ν increases. Note that in this type of model \dot{G}_N/G_N is positive and is of order of Hubble constant. This is in contrast to more conventional models where \dot{G}_N/G_N is negative and is order of $1/\omega$ times the Hubble constant [21]. Thus this type of model predicts a time varying Newton's gravitational constant which is more amenable to being tested by experiment.

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